

Better AOQ Formulas

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ABSTRACT:

The classic formula for AOQ calculates an accurate value for Type-B sampling but an inaccurate, low value for Type-A sampling. That inaccuracy is most noticeable when the lot sizes being sampled are small or when the AOQ being calculated is also the AOQL. A new Type-A-sampling AOQ formula has been developed that provides accurate AOQ's (and thereby accurate AOQL's) for any size lot. For Type-B sampling, a new formula has been developed that outputs an AOQ that is numerically identical to that outputted by the classic formula but whose components provide useful information that is unavailable from the classic formula's components. Based upon those new component formulas, student-friendly equations have been developed that output the various AOQ values applicable to various practices in incoming QC inspection.

INTRODUCTION:

One way to implement "risk management" in IQC (the incoming quality control department) is to focus on the AOQ's or AOQL's of IQC's sampling plans. AOQ (average outgoing quality) is typically expressed in units of percent defective. The Statistics Division of the American Society for Quality (ASQ) has defined AOQ as "the expected quality level of outgoing product following the use of an acceptance sampling plan for a given value of incoming product quality" (ASQC, p. 48). In this article, the term "incoming product" means parts, components, or finished goods that have been received for inspection by IQC (the incoming quality control department) either from another department or from an external supplier; and the term "outgoing product" means what is released by IQC for use in production. When a wide range of incoming product percent defective is plotted vs. the corresponding AOQ's, as shown in Figure 1, the peak of the curve identifies the AOQL (average outgoing quality limit), which is the worst-case scenario for outgoing product quality.

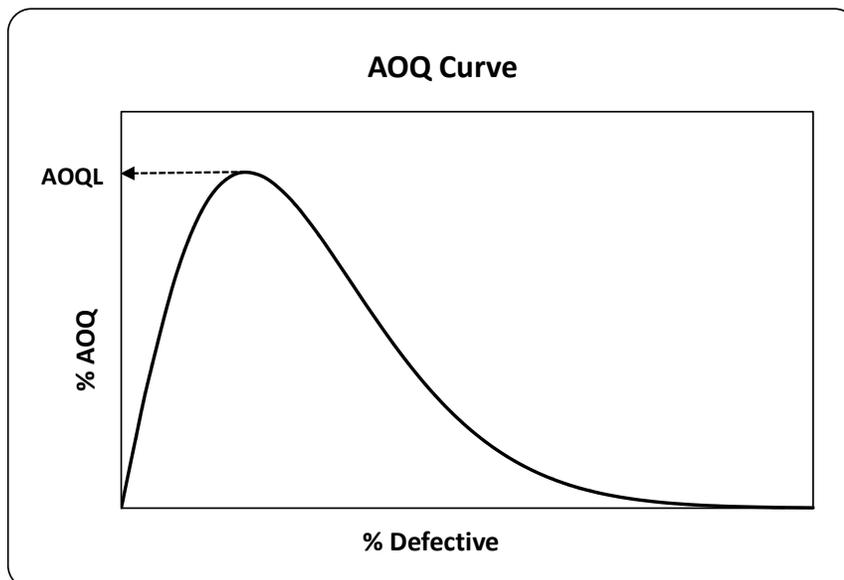


Figure 1: Example of an AOQ curve

The Y-axis value (%AOQ) for a single point on an AOQ curve is a function of multiple variables, among which are P_a (i.e., the acceptance sampling plan's "probability of acceptance") and P_d (i.e., the corresponding X-axis %defective). The calculation of P_a and the explanation of P_d depend upon whether the sampling is Type-A or Type-B. The purpose of Type-A sampling is "to accept or reject the immediate *lot* of product at hand," whereas the purpose of Type-B sampling is "to determine if the *process* that produced the product at hand was within acceptable limits" (Juran and Gryna, p. 25.4; italics were added).

P_a -values for Type-A-sampling relate to "accepting a lot as a function of *lot* quality"; and therefore "for various values of *lot* fraction defective", the corresponding probabilities are "computed from the hypergeometric distribution". In contrast, P_a -values for Type-B-sampling relate to "accepting a lot as a function of *product* quality"; and therefore "for various values of *product* fraction defective", the corresponding probabilities are "computed from the binomial". Type-A sampling applies when production is sporadic or when the main concern is to control the "Consumer's Risk...of accepting a lot having...the lot tolerance fraction defective". Type-B sampling applies when production is relatively constant or when the main concern is to control the "Producer's Risk" of rejecting "product having a fraction defective...equal to the process average fraction defective". (Dodge and Romig, 1959, pp. 56-57)

For the purpose of calculating the Y-axis value for a single point on an AOQ curve, Type-A sampling considers each incoming lot to be a "*finite universe* or *lot* having the stated value of fraction defective"; i.e., each incoming *lot* has the X-axis %defective corresponding to that AOQ point. Therefore, for the purpose of calculating a single AOQ-curve point, an AOQ calculation for Type-A sampling considers incoming lots to *all* have the *same* P_d . In contrast, for the purpose of calculating the Y-axis value for a single AOQ-curve point, Type-B sampling considers each incoming lot to have been obtained by "sampling from an *infinite universe* or *product* having a stated value of fraction defective"; i.e., incoming lots are random samples from a *process* that has the X-axis %defective corresponding to that AOQ point, and therefore incoming lots have a "binomial distribution of lot quality values". (Dodge and Romig, 1959. p. 56)

THE CLASSIC AOQ FORMULA

The classic AOQ formula is shown below as Equation 1. This was available implicitly by at least 1941 when Dodge and Romig published equations whose algebraic combination yielded it (Dodge and Romig, 1941, equations 8 and 12); subsequent authors published the classic formula explicitly (e.g., Western Electric, p. 251; Juran and Gryna, p. 25.13; and NIST, on the web-page that contains the definition of AOQ).

$$\text{Equation 1: } \quad \text{Classic AOQ} = [(P_d \cdot P_a) (N - n)] / N$$

where IQC inspection efficiency is 100%, and

- P_d = percent defective in each lot (Type-A sampling) or the average percent defective in a series of lots (Type-B sampling) received for IQC; in either case, P_d is the AOQ-curve X-axis %defective for which a Y-axis AOQ value is sought.
- P_a = probability of IQC accepting lots or product that have a given P_d , where probability is calculated using either the hypergeometric distribution (Type-A sampling) or the binomial distribution (Type-B sampling).
- N = size of finite lots received for IQC (this is a constant for all calculations for a given AOQ curve).
- n = size of samples from each lot received for IQC (this is a constant for all calculations for a given AOQ curve).

Equation 1 can be algebraically rearranged into the following:

$$\text{Equation 2: } \quad \text{Classic AOQ, rearranged} = [(N \cdot P_d \cdot P_a) - (n \cdot P_d \cdot P_a)] / N$$

The classic AOQ formula assumes that IQC has taken the following classic disposition practice *before* releasing a lot for use in production:

- all defectives found in samples from accepted lots are either repaired or replaced with good product
- all defectives found in rejected lots are either repaired or replaced with good product (this includes all defectives found during inspection of the sample and found during inspection of the remaining portion of the lot; all rejected lots must be 100% inspected for Equations 1 and 2 to apply).

There are many variants of the classic AOQ formula that apply either when lot-size is extremely large compared to sample size, or when inspection efficiency is not 100%, or when the requirements to repair or replace are not fully met. In this author's experience, virtually no companies use the classic disposition practice in IQC but rather use alternative practices such as not replacing the discarded defectives.

The classic formula has been called "the most common expression for the AOQ" (Beainy and Case, p. 6), and it has been applied to both Type-A and Type-B sampling. Published examples of its application to Type-A sampling include: "[*the classic formula outputs the*] AOQ for specific fractions nonconforming of submitted *lots*" (Pyzdek, p. 295); and "if all *lots* come in [*to IQC*] with a defect level of *exactly* [*Pd*], ...the AOQ can easily be shown to be [*the classic formula*]" (NIST, on the web-page that contains the definition of AOQ). Published examples of the classic formula's application to Type-B sampling include: "The meaning of the term AOQL [*as calculated based upon the classic AOQ formula*] can be illustrated as follows...when *product* is 2% defective..." (Western Electric, p. 249); and the Dodge-Romig AOQL sampling tables, which are segregated by "*Process Average % [defective]*" (Dodge and Romig, 1941). [All italics and bracketed text were added.]

It is important to note that all defective product that remains in a series of outgoing lots exists in the non-sample (and therefore not inspected) portions of the *accepted* lots; all other defective product would have already been removed during inspection of samples and during inspection of the non-sample portions of rejected lots (assuming that inspection accuracy is perfect and that all rejected lots have their non-sample portion 100% inspected). The numerator of the classic AOQ formula therefore represents the defectives in all incoming lots that will be IQC "accepted", minus the defectives that will be removed from all IQC samples from those accepted lots; the denominator represents all items (defective and non-defective) in all incoming lots received for IQC inspection.

TYPE-A SAMPLING

The accuracy of the classic AOQ formula for Type-A sampling can be assessed by comparing its output to that obtained by counting the defectives in a series of *virtual* outgoing lots after they have undergone single-sampling. Such virtual comparisons are shown in Examples 1 and 2, for large and small lots respectively, over the course of 52 lots. In these examples, the classic AOQ formula is found to be accurate to almost 4 significant digits when lot size is large, but accurate to only 1 significant digit (after rounding) when lot size is small; the same AOQ's would be obtained if a million lots were used instead of 52.

52	= L = number of lots in the series of lots analyzed in this virtual example	
100,000	= N = lot size	
10%	= Pd = percent defective in every lot in the series (Type-A sampling)	
30	= n = sample size	
3	= Ac = maximum number of defectives allowed in an accepted sample	
i = number of defectives found in a sample	Pi = hypergeometric probability of finding exactly " i " defectives in a sample (Type-A sampling)	i x Pi x L
0	0.04237	0.0000
1	0.14128	7.3466
2	0.22767	23.6774
3	0.23612	36.8352
Totals =	0.64744 = Pa	68
Total number of defectives in all Accepted Lots before IQC = Pd x Pa x N x L =		336,670
Total number of defectives in all Accepted Lots after removing defectives found in samples = (Pd x Pa x N x L) - sum(i x Pi x L) =		336,602
If all found defectives are replaced by good parts, then the number of parts that IQC releases for use = N x L =		5,200,000
Virtual Counting Method AOQ = 336602 / 5200000 =		6.4731%
Classic AOQ = Pd x Pa x (N - n) / N =		6.4725%

Example 1: Virtual-counting AOQ vs. Classic AOQ, with Type-A sampling of large lots.
All calculations by MS Excel; intermediate calculation significant digits not rounded.

52	= L = number of lots in the series of lots analyzed in this virtual example	
100	= N = lot size	
10%	= Pd = percent defective in every lot in the series (Type-A sampling)	
30	= n = sample size	
3	= Ac = maximum number of defectives allowed in an accepted sample	
i = number of defectives found in a sample	Pi = hypergeometric probability of finding exactly " i " defectives in a sample (Type-A sampling)	i x Pi x L
0	0.02292	0.0000
1	0.11271	5.8608
2	0.23723	24.6721
3	0.28116	43.8615
Totals =	0.65402 = Pa	74
Total number of defectives in all Accepted Lots before IQC = Pd x Pa x N x L =		340
Total number of defectives in all Accepted Lots after removing defectives found in samples = (Pd x Pa x N x L) – sum(i x Pi x L) =		266
If all found defectives are replaced by good parts, then the number of parts that IQC releases for use = N x L =		5,200
Virtual Counting Method AOQ = 266 / 5200 =		5.1095%
Classic AOQ = Pd x Pa x (N – n) / N =		4.5781%

Example 2: Virtual-counting AOQ vs. Classic AOQ, with Type-A sampling of small lots.
All calculations by MS Excel; intermediate calculation significant digits not rounded.

The accuracy of the classic formula can be improved for Type-A sampling by removing "n x Pd x Pa" from Equation 2 and replacing it with the formula in Equation 3, thereby creating Equation 4:

Equation 3: Exact Correction for Defectives in Accepted Sample (Type A sampling) = $\sum_{i=0}^{i=Ac} (i \cdot Pi)$

Equation 4: Exact AOQ (Type A sampling) = $[(N \times Pd \times Pa) - \sum_{i=0}^{i=Ac} (i \cdot Pi)] / N$

where i, Pi, and Pa are as defined in Examples 1 and 2, and

Ac = IQC inspection acceptance number (a.k.a., "C"). If an IQC inspection sample is found to have Ac or less defectives, the non-defective members in it are "accepted" into production along with all of the uninspected (i.e., non-sample) portion of the finite size lot from which the sample came; if an IQC inspection sample is found to have more than Ac defectives, the sample and lot are rejected and subjected to either 100% inspection, return to supplier, or other possible dispositions.

As seen in Tables 1a and 1b, this new AOQ formula calculates values that are exactly equal to the virtual-counting AOQ for both large and small lots. The proposed name for the output of such a calculation is an "Exact AOQ."

Lot Size	10	20	30	100	1,000	10,000	100,000
Assumed % defective	10 %						
Sample Size	3	5	8	13	80	125	315
Ac	0	1	2	3	7	14	21
Pa	0.7000	0.9474	0.9862	0.9762	0.4400	0.7340	0.0252
"Exact correction"	0.0000	0.3947	0.7586	1.2014	2.5041	8.0512	0.4959
Classic AOQ %	4.9000	7.1053	7.2322	8.4926	4.0480	7.2483	0.2509
Virtual Counting AOQ %	7.0000	7.5000	7.3333	8.5602	4.1496	7.2595	0.2512
Exact AOQ %	7.0000	7.5000	7.3333	8.5602	4.1496	7.2595	0.2512

Table 1a: Comparison of AOQ's calculated for Type-A sampling by different methods. Sampling plans taken from ANSI/ASQC Z1.4, Single Sampling Plans for a variety of AQL's. The "Virtual Counting AOQ" method was demonstrated in Examples 1 and 2.

Lot Size	10	20	30	100	1,000	10,000	100,000
Assumed % defective	10 %						
Sample Size	3	3	5	6	15	29	74
Ac	0						
Pa	0.7000	0.7158	0.5665	0.5223	0.2035	0.0469	0.0004
"Exact correction"	0.0000						
Classic AOQ %	4.9000	6.0842	4.7209	4.9097	2.0043	0.4675	0.0041
Virtual Counting AOQ %	7.0000	7.1579	5.6650	5.2230	2.0348	0.4689	0.0041
Exact AOQ %	7.0000	7.1579	5.6650	5.2230	2.0348	0.4689	0.0041

Table 1b: Comparison of AOQ's calculated for Type-A sampling by different methods. Sampling plans taken from Squeglia's Zero Acceptance Number Sampling Plans, for a variety of AQL's. The "Virtual Counting AOQ" method was demonstrated in Examples 1 and 2.

As seen in Figure 2, the difference between the Exact and Classic AOQ curves is greatest at the peak of the curves, where AOQ equals AOQL.

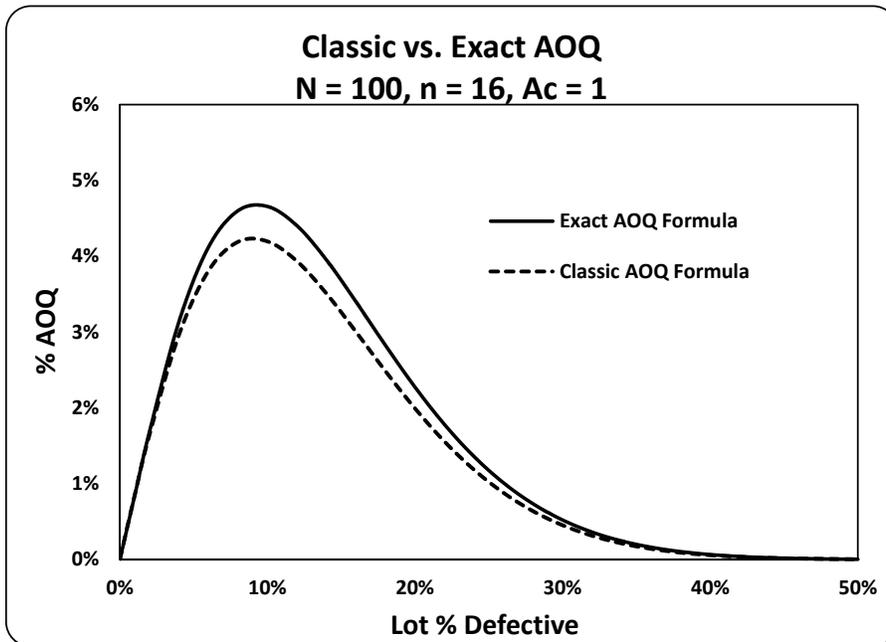


Figure 2: Example of Classic vs. Exact AOQ curves for Type-A Sampling.

TYPE-B SAMPLING

An alternate formula for calculation of AOQ for Type-B sampling has been developed (Equation 5, below). Although it is mathematically identical to the classic formula (i.e., Equations 1, 2, and 5 output the identical AOQ value), it provides intermediate components (aLd and aSd) that, when divided by Pa (the probability of acceptance), provide useful information that is not available from the corresponding classic components, as shown in Examples 3 and 4.

Equation 5: Alternate Formula for the (Eq. 2) Classic AOQ (Type B sampling) = (aLd – aSd) / N

Equation 6: $aLd = \sum_{k=0}^{k=N} \sum_{i=0}^{i=Ac} (k \cdot Pk \cdot Pik),$

where aLd / Pa = average number of defectives that existed in *to-be-accepted lots* before sampling for IQC; the corresponding classic component (N x Pd x Pa), when divided by Pa, equals N x Pd = the average number of defectives in *average* lots (approved plus rejected) before sampling for IQC,

Equation 7: $aSd = \sum_{k=0}^{k=N} \sum_{i=0}^{i=Ac} (i \cdot Pk \cdot Pik),$

where aSd / Pa = average number of defectives found in IQC *samples* from lots subsequently *accepted*; the corresponding classic component (n x Pd x Pa), when divided by Pa, equals n x Pd = the average number of defectives found in IQC samples from an *average* lot (i.e., both approved and rejected lots),

For Equations 6 and 7, N, i, and Ac have been defined above, and

k = number of defectives in a lot, prior to IQC sampling

Pk = binomial probability that the finite size lot (= N) being IQC-inspected contains exactly k defectives, assuming it was drawn randomly from an infinite-size lot having Pd percent defective
= binomdist (k , N , Pd , false), using a Microsoft Excel function formula.

Pik = hypergeometric probability that the IQC sample (= n) contains exactly i defectives, assuming that it was drawn randomly from the finite-size lot (= N) being IQC-inspected that contains exactly k defectives
= hypgeomdist (i , n , k , N), using a Microsoft Excel function formula.

Examples 3 and 4 provide comparisons of the numerator components of Equation 2 vs. those of Equation 5, assuming Type-B sampling. The outputs of the classic and alternate AOQ formula numerators are identical (both equal 2.116198062 in Example 3, and both equal 0.947027862 in Example 4).

EXAMPLE 3: Type-B sampling, from ANSI Z1.4, Single Sample, Normal, General Inspection Level II, AQL 10%:

$$0.10 = Pd = 10\%$$

$$30 = \text{Lot Size}$$

$$8 = \text{Sample Size}$$

$$2 = Ac$$

$$0.962 = Pa = \text{fraction of lots that are accepted} = \text{BINOMDIST}(2, 8, 0.10, \text{true}), \text{ using Microsoft Excel.}$$

Classic Formula (Equation 2):

$$N \times Pd \times Pa = 30 \times 0.100 \times 0.962 = 2.886 \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives that existed in an average lot (accepted and rejected, combined) prior to IQC sampling} = 2.886 / 0.962 = 3.000$$

$$n \times Pd \times Pa = 8 \times 0.100 \times 0.962 = 0.770 \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives that were found samples from those average lots} = 0.770 / 0.962 = 0.800$$

$$2.886 - 0.770 = \mathbf{2.116} \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives remaining in the uninspected portion of an accepted lot} = 2.116 / 0.962 = 2.200$$

Alternate Formula (Equation 5):

$$\text{Sum of } (k \times Pk \times Pik) = 2.796 \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives that existed in an accepted lot prior to IQC sampling} = 2.796 / 0.962 = 2.907$$

$$\text{Sum of } (i \times Pk \times Pik) = 0.680 \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives that were found in a sample from an accepted lot} = 0.680 / 0.962 = 0.707$$

$$2.796 - 0.680 = \mathbf{2.116} \quad \ll \text{ this is the same numerator value as was generated by Equation 2.}$$

EXAMPLE 4: Type-B sampling, from ANSI Z1.4, Single Sample, Normal, General Inspection Level II, AQL 1.5%:

$$0.10 = Pd = 10\%$$

$$30 = \text{Lot Size}$$

$$8 = \text{Sample Size}$$

$$0 = Ac$$

$$0.430 = Pa = \text{fraction of lots that are accepted} = \text{BINOMDIST}(0, 8, 0.10, \text{true}), \text{ using Microsoft Excel.}$$

Classic Formula (Equation 2):

$$N \times Pd \times Pa = 30 \times 0.100 \times 0.430 = 1.290 \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives that existed in an average lot (accepted and rejected, combined) prior to IQC sampling} = 1.291 / 0.430 = 3.000.$$

$$n \times Pd \times Pa = 8 \times 0.100 \times 0.430 = 0.344 \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives that were found samples from those average lots} = 0.344 / 0.430 = 0.800.$$

$$1.291 - 0.344 = \mathbf{0.947} \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives remaining in the uninspected portion of an accepted lot} = 0.947 / 0.430 = 2.200.$$

Alternate Formula (Equation 5):

$$\text{Sum of } (k \times Pk \times Pik) = 0.947 \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives that existed in an accepted lot prior to IQC sampling} = 0.947 / 0.430 = 2.200.$$

$$\text{Sum of } (i \times Pk \times Pik) = 0.000 \quad \ll \text{ when divided by } Pa, \text{ this value equals the average number of defectives that were found in a sample from an accepted lot} = 0.000 / 0.430 = 0.000.$$

$$0.947 - 0.000 = \mathbf{0.947} \quad \ll \text{ this is the same numerator value as was generated by Equation 2.}$$

AOQ FORMULA VARIANTS

ASQ advises that the "computational formula for the AOQ depends on whether or not the [IQC] sample items and the lots not-accepted [= *rejected*] are discarded, and whether or not variant [= *defective*] units detected during sampling or 100 percent inspection are replaced by acceptable units" (ASQC, p. 48; bracketed text was added). Many different AOQ formulas have long been available that apply to various IQC disposition practices (e.g., see Beainy & Case, Case *et. al.*, Wortham & Mogg, and Taylor); Tables 2, 3, and 4 , and Figures 3a and 3b provide a sampling of relevant AOQ formula variants and their resulting AOQ values.

It is important to notice the wide variation in achieved AOQ that is possible, caused solely by differences in disposition practices for defectives (e.g., 5.8% vs. 9.1%, in Table 4, Type-B), and to note that the AOQ based upon classic practice (i.e., Type-I in Table 2) is the only one provided in the classic Dodge-Romig tables, and in ANSI-Z1.4 (and in its ISO equivalent, 2859-1), and in Squeglia's Zero Acceptance Number Sampling Plans, despite the fact that such an IQC practice is virtually never used in today's companies. In this author's experience of auditing IQC at more than 200 companies, by far the most common IQC disposition practice today for non-destructive test/inspection is Type III.

AOQ Type	IQC disposition practices that affect AOQ values			
	Accepted Lots		Rejected Lots	
	sample	remainder	sample	remainder
I	defectives discarded and replaced with good parts; all good parts are then released for use in production	all are released for use in production	100% inspected; defectives discarded and replaced with good parts; all good parts are then released for use in production	
II				
III	defectives discarded but NOT replaced; all good parts are then released for use in production		100% inspected; defectives discarded but NOT replaced; all good parts are then released for use in production	
IV			all lots are discarded but NOT replaced	

Table 2: AOQ Types defined by means of IQC disposition practices.

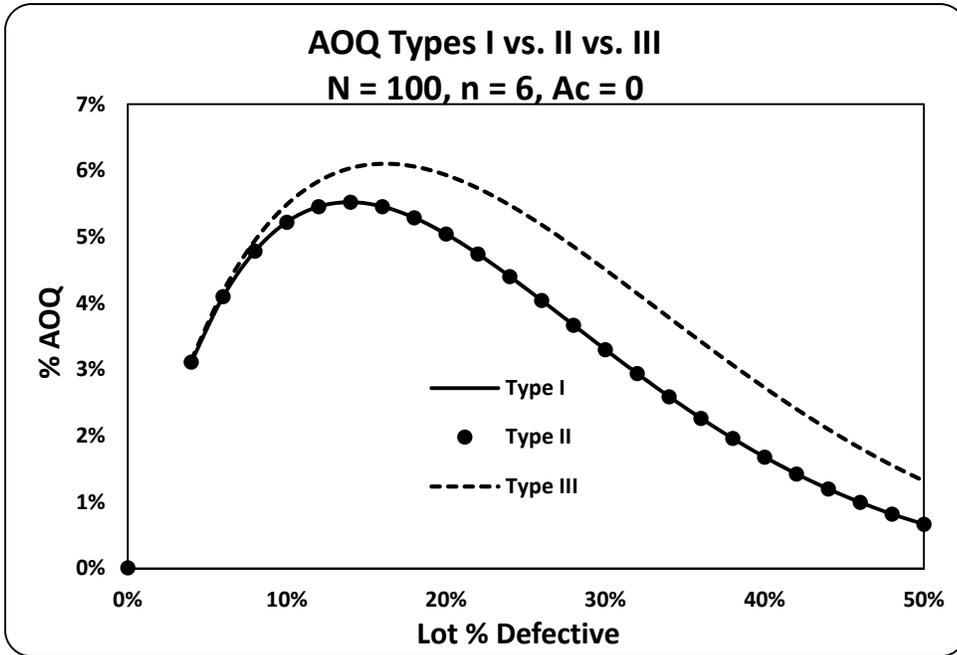


Figure 3a: Example of Exact AOQ curves for AOQ Types I, II, and III, for Type-A Sampling.

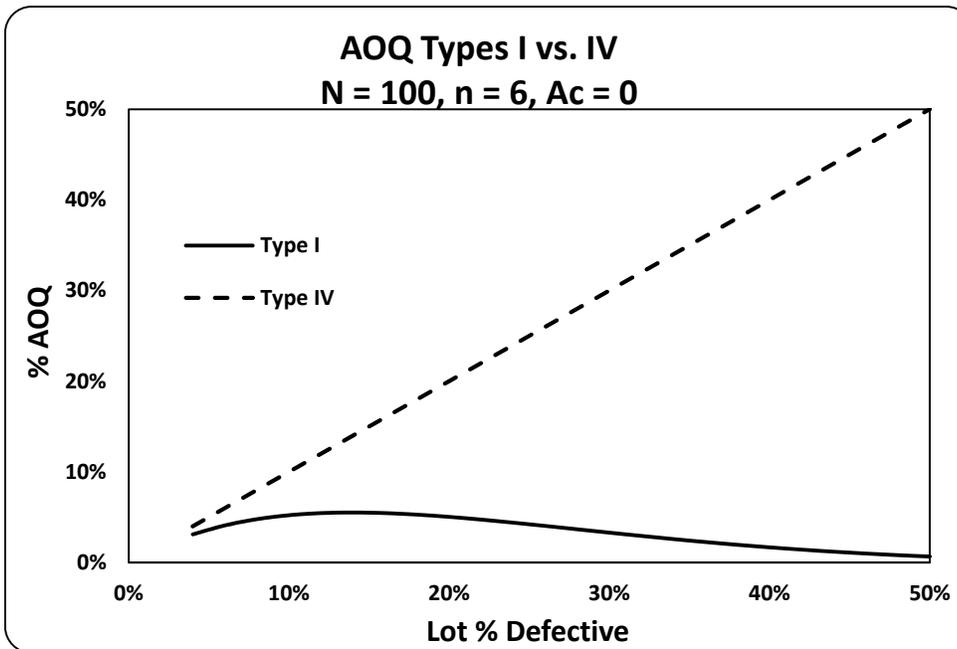


Figure 3b: Example of Exact AOQ curves for AOQ Types I and IV, for Type-A Sampling.

	TYPE-A SAMPLING	TYPE-B SAMPLING
AOQ calculations		
Type-I	$[(N \times Pd \times Pah) - Z] / [N]$	$[(N \times Pd \times Pab) - (n \times Pd \times Pab)] / [N]$ = "classic formula" or $[aLd - aSd] / [N]$ = "alternate formula" (see *** below)
Type-II	$[(N \times Pd \times Pah) - Z] / [N - Z]$	$[(N \times Pd \times Pab) - (n \times Pd \times Pab)] / [N - aSd]$
Type-III	$[(N \times Pd \times Pah) - Z] / [N - Z - (N \times Pd \times (1 - Pah))]$ or $[(N \times Pd \times Pah) - Z] / [N \times (1 - Pd) + (N \times Pd \times Pah - Z)]$	$[(N \times Pd \times Pab) - (n \times Pd \times Pab)] / [N - aSd - (N \times Pd - aLd)]$ or $[(N \times Pd \times Pab) - (n \times Pd \times Pab)] / [N \times (1 - Pd) + (aLd - aSd)]$
Type-IV	$[(N \times Pd \times Pah) - Z] / [N - Z - (N \times (1 - Pah))]$ or $[(N \times Pd \times Pah) - Z] / [N \times Pah - Z]$	$[(N \times Pd \times Pab) - (n \times Pd \times Pab)] / [N - aSd - (N \times (1 - Pab))]$ or $[(N \times Pd \times Pab) - (n \times Pd \times Pab)] / [N \times Pab - aSd]$
Average number of defectives per lot, prior to IQC sampling		
average lot	N x Pd	N x Pd
will-be-accepted lot		aLd / Pab
will-be-rejected lot		$(N \times Pd - aLd) / (1 - Pab)$
Average number of defectives per IQC sample, prior to inspection		
average lot	n x Pd	n x Pd
will-be-accepted lot	Z / Pah	aSd / Pab
will-be-rejected lot	$(n \times Pd - Z) / (1 - Pah)$	$(n \times Pd - aSd) / (1 - Pab)$
Average number of defectives that remains in a lot after the IQC sample is removed		
average lot	$(N \times Pd) - (n \times Pd)$	$(N \times Pd) - (n \times Pd)$
will-be-accepted lot	$(N \times Pd) - (Z / Pah)$	
will-be-rejected lot	$(N \times Pd) - [(n \times Pd - Z) / (1 - Pah)]$	

Table 3. Formulas for calculation of a few types of AOQ's and related values; the definitions of symbols used in the formulas are listed below.

- N, Pd, and n are defined below Equation 1
- Z is defined by Equation 3
- aLd is defined by Equation 6; *** this does *not* equal $N \times Pd \times Pab$
- aSd is defined by Equation 7; *** this does *not* equal $n \times Pd \times Pab$
- Pah is a sampling plan's "probability of acceptance" calculated based upon the hypergeometric distribution
- Pab is a sampling plan's "probability of acceptance" calculated based upon the binomial distribution

	TYPE-A SAMPLING Pa = 0.64889	TYPE-B SAMPLING Pa = 0.6474
AOQ Type-I	6.0415%	5.8270%
AOQ Type-II	6.0687%	5.8524%
AOQ Type-III	6.2905%	6.0807%
AOQ Type-IV	9.3752%	9.0609%
number of defectives in average lot (prior to sampling)	30.0000	30.0000
number of defectives in average accepted lot (prior to sampling)	30.0000	29.0154
number of defectives in average rejected lot (prior to sampling)	30.0000	31.8080
number of defectives found in average lot sample	3.0000	3.0000
number of defectives found in average accepted-lot sample	2.0683	2.0154
number of defectives found in average rejected-lot sample	4.7218	4.8080
number of defectives in non-sample portion of average lot	27.0000	27.0000
number of defectives in non-sample portion of average accepted lot	27.9317	27.0000
number of defectives in non-sample portion of average rejected lot (prior to 100% inspection)	25.2782	27.0000

Table 4. Examples of values resulting from Table 3's formulas, when used with the following 5% AQL plan: Pd = 10%, N = 300, n = 30, and Ac = 3

For Type-A sampling, all lots are 10% defective.

For Type-B sampling, lot quality varies per the binomial distribution but averages 10% defective.

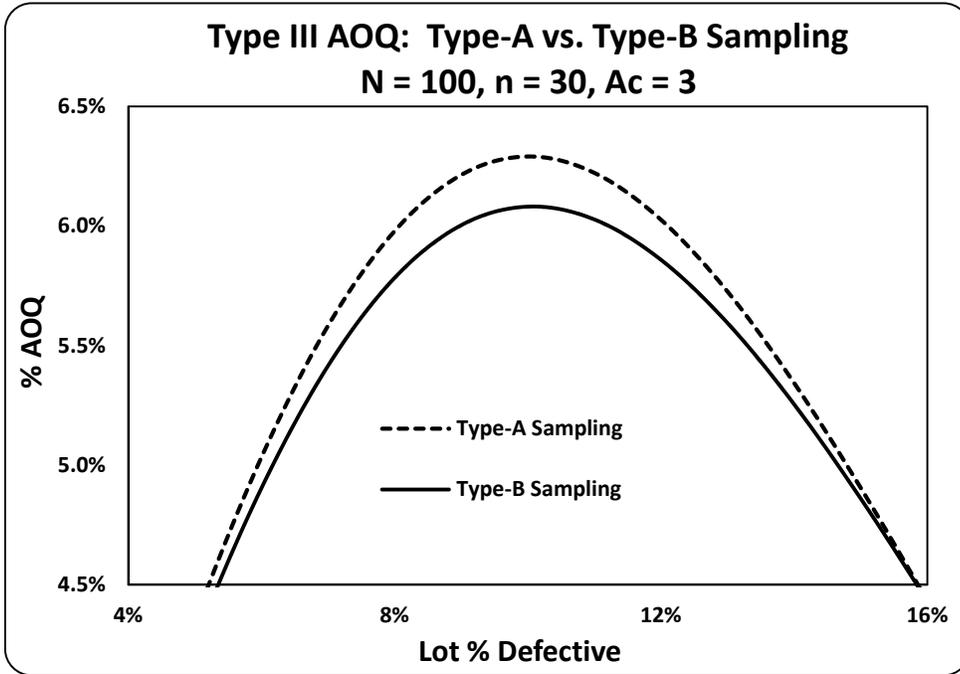


Figure 4: Example of differences in AOQL's (i.e., peaks of AOQ curves) for Type-A vs. Type-B Sampling.

SUMMARY AND RECOMMENDATIONS:

When used with Type-A sampling, the classic AOQ formula produces inaccurate, low values, especially for small lots or for the AOQ that equates to the AOQL. Therefore neither the classic formula nor its algebraic rearrangement should be used for Type-A sampling when a high level of accuracy is desired; instead, the "Exact AOQ" formulas introduced in this article should be used because they provide exact AOQ values for any lot size and for any point on the AOQ curve.

When used with Type-B sampling, the classic formula for AOQ does provide an accurate value for any lot size and any point on the AOQL curve; however, the alternate formula introduced in this article not only outputs the identical AOQ value as does the classic formula, but also replaces the classic intermediate-calculation components with alternate ones that may aid in teaching the meaning of AOQ to new students of the topic.

When a company's purpose for sampling is Type-B (i.e., to determine if the *process* that produced the product at hand was within acceptable limits), but company practice is closer to being Type-A (i.e., to accept or reject the immediate *lot* of product at hand), a more conservative approach to risk management is to calculate AOQL's under the assumption of Type-A rather than Type-B, because AOQL's for Type-A are larger than those for Type-B, as shown in Figure 4; that is, Type-A is the worst-case scenario.

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