

**WHY DOES RELIABILITY PLOTTING TYPICALLY PROVIDE A HIGHER PROCESS CAPABILITY RESULT  
(THAT IS, %RELIABILITY RESULT) THAN DOES A K-TABLE?**

**SUMMARY STATEMENT:**

Reliability Plotting (RP) typically outputs a larger %Reliability than does a K-table because RP uses one additional piece of information that K-tables do not; that piece of information (the "Standard Error of the Estimate") is analogous to a Correlation Coefficient. Also, when QC specifications are 2-sided, RP automatically adds a correction when a sample average is not perfectly centered between the upper and lower specifications, whereas K-tables do not; such a correction results in a larger %Reliability.

**DETAILED STATEMENT:**

It is often desired to estimate what % of a lot or population is inside and outside the QC specifications. For a given lot, the % inside the specifications (a.k.a., %-in-spec) is referred to as a %Reliability. When describing the process that produced the lot, the %-in-spec is referred to either as %Reliability or as Process Capability.

If a sample from a population indicates that the population has a "Normal" distribution, then Normal Distribution "Z-tables" (see Fig. 1) can be used to calculate a %Reliability. However, such a result is a *statistic* without any associated confidence level, and therefore it is not a useful estimate of the corresponding true population *parameter*. To estimate that parameter, what is typically done is to calculate the lower 1-sided 95% confidence limit for the sample statistic %Reliability. For example, if a sample's Z-score is 2.576, then a Z-table indicates that the %-in-spec (for 2-sided specs) is 99% (i.e., only 1% out-of-spec); however, using additional methods, it might be determined that the lower 1-sided 95% confidence limit for the population parameter is 96% for a large sample or 76% for a small sample.

**NORMAL AREAS (A-4)**

<i>z</i>	<i>Cum.</i>	<i>Central</i>	<i>Two tail</i>	<i>Upper tail</i>
1.000	.841	.683	.317	.159
1.500	.933	.866	.134	.067
1.645	.950	.900	.100	.050
1.960	.975	.950	.050	.025
2.326	.990	.980	.020	.010
2.576	.995	.990	.010	.005
3.09	.999	.998	.002	.001
3.29	.9995	.999	.001	.0005
3.72	.9999	.9998	.0002	.0001

**Fig. 1: Z-table**

Tables have been created that cross-reference Z-scores vs. %Confidence vs. population parameter %Reliability. Such tables are called by names such as "Statistical Tolerance Limit Factors K for a Normal Distribution", or "K-table" for short. For example, if a sample average and standard deviation are 101 and 10 respectively, and if the 2-sided QC specs are 76 and 124, then 2.30 is the appropriate Z-score:

$$Z\text{-score} = (124 - 101) / 10 = 2.30 \quad (\text{when using K-tables, this is called the "observed K"})$$

Use of a Z-table allows a conclusion that a score of 2.30 equals 97.9% Reliability vs. a 2-sided QC spec. However, as discussed above, that 97.9% is only a statistic without an associated "confidence" level. On

the other hand, a K-table (see Fig. 2) allows a conclusion that an observed K of 2.30 equals a population parameter %Reliability of 92.3% for n=30 and of 77.2% for n=8, at 95% confidence (as determined by a 4th-order polynomial interpolation of the values of K and %Reliability found in the K-table).

Confidence = 95 %							
<i>N</i>	<i>p</i> = 0,750	<i>p</i> = 0,900	<i>p</i> = 0,950	<i>p</i> = 0,975	<i>p</i> = 0,990	<i>p</i> = 0,995	<i>p</i> = 0,999
2	22,383	31,092	36,519	41,308	46,944	50,813	58,844
3	5,937	8,306	9,789	11,101	12,647	13,710	15,920
4	3,818	5,368	6,341	7,203	8,221	8,921	10,377
5	3,041	4,291	5,077	5,774	6,598	7,165	8,345
6	2,638	3,733	4,422	5,034	5,758	6,256	7,294
7	2,391	3,390	4,020	4,579	5,241	5,697	6,647
8	2,223	3,156	3,746	4,269	4,889	5,316	6,206
9	2,101	2,986	3,546	4,044	4,633	5,039	5,885
10	2,008	2,856	3,393	3,871	4,437	4,827	5,640
28	1,519	2,170	2,585	2,954	3,393	3,696	4,330
29	1,510	2,157	2,569	2,937	3,373	3,675	4,304
30	1,501	2,145	2,555	2,921	3,355	3,654	4,281

Fig. 2: Two-sided Normal Tolerance K-table

An alternative to K-tables is a linear-regression-based method called Reliability Plotting (see Fig. 3, and see Tobias and Trindade, *Applied Reliability*, 3rd edition). When this method is augmented with classic linear regression confidence limits for the mean of a point on a line, the combined method typically outputs a higher estimate for the population parameter %Reliability than does a K-table, for the same data set. There are two reasons for that difference, both of which are discussed below.

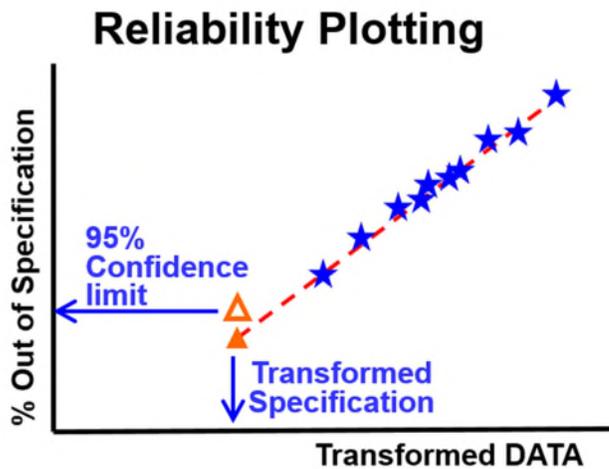


Fig. 3: Generic Example of Reliability Plot

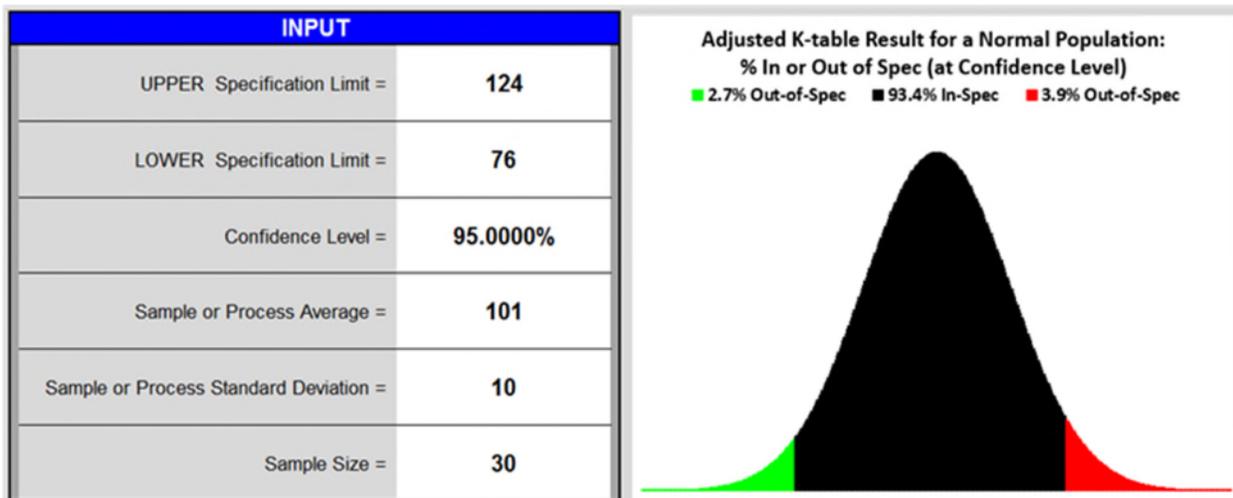
The first reason applies no matter whether the QC specs are 1-sided or 2-sided. This reason involves the number of preliminary statistics used to calculate the final result. K-table results are based upon the inputted Z-score, which is based upon these statistics:

- Sample Average
- Sample Standard Deviation
- Range (from Sample Average to the *closer* QC-spec).

Reliability Plotting's results are based on all three of those statistics plus one additional. The additional statistic is called the "Standard Error of the Estimate" (a.k.a. SEE), which is an output of the linear regression calculations used in Reliability Plotting. An SEE provides information similar to what is provided by a linear regression Correlation Coefficient; that additional information allows Reliability Plotting to calculate a higher %Reliability than is possible with K-tables.

The second reason applies only when QC specs are 2-sided. This reason involves the fact that K-tables classically assume that the sample average is exactly midway between the upper and lower QC specs. When using K-tables classically, the %Reliability is determined in reference to the *closer* QC spec, but no correction is made for off-center sample averages. In the n=30 example above, K-table %Reliability was 92.3% at 95% confidence, based upon the observed K of 2.30, which is the distance (in standard deviation units) from the sample average to the *upper* QC spec (which is the closer). The distance from the sample average to the *lower* QC spec is 2.50 standard deviation units (i.e.,  $2.50 = (101 - 76) / 10$ ). If we were to *incorrectly* assume that 76 is the "closer" spec, then that 2.50 would result in a 95% confidence K-table %Reliability of 94.5%. Neither that 94.5% nor the 92.3% is correct for the population from which the sample of 30 was taken. The correct result is obtained by adding those 2 results and multiplying by 50%; in this case, the correct result is:

$$(94.5\% + 92.3\%) \times 50\% = 93.4\%.$$



**Fig. 4: %In-Specification Result using K-tables Adjusted (i.e., Corrected) for Off-center Mean**

K-tables cannot make such a correction automatically; it must be done manually or by using spreadsheets (e.g., Worksheet P, in "Reliability Statistics Basics", found at [www.johnzorich.com](http://www.johnzorich.com)). On the other hand, Reliability Plotting makes such a correction automatically (e.g., Worksheet N in that spreadsheet). Thus, when calculating %Reliability vs. a 2-sided QC spec, Reliability Plotting typically outputs a higher %Reliability than does a classic, uncorrected K-table analysis.